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Electromagnetic string fluid in rolling tachyon

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Abstract: We study Born-Infeld type effective action for unstable D3-brane system including a tachyon and an abelian gauge field, and find the rolling tachyon with constant electric and magnetic fields as the most general homogeneous solution. Tachyonic vacua are characterized by magnitudes of the electric and magnetic fields and the angle between them. Analysis of small fluctuations in this background shows that the obtained configuration may be interpreted as a fluid consisting of string-like objects carrying electric and magnetic fields. They are stretched along one direction and the rolling tachyon move in a perpendicular direction to the strings. Direction of the propagating waves coincides with that of strings with velocity equal to electric field.

Keywords: Brane Dynamics in Gauge Theories, Tachyon Condensation.

Contents

1. Introduction

Instability of an unstable D-brane or a pair of D-brane-anti-D-brane is manifested by a tachyonic mode and, in the context of effective field theory, decay of the unstable brane is depicted by condensation of tachyon [1]. As the tachyon approaches its true vacuum where brane tension is dissipated away, open string degrees of freedom disappear and correspondingly perturbative plane-wave type excitations are completely absent in the effective field theory. The classical decay process is described by the so-called rolling tachyons which are constructed as a family of spatially-homogeneous but time-dependent classical solutions in open string theory [2, 3].

This rolling tachyon has been mainly applied to various topics in cosmology in the very early universe, e.g., inflation, dark matter, and reheating [4]–[7]. In tachyon cosmology or other applications, inclusion of bosonic string degrees looks indispensable, e.g., gauge field, graviton, dilaton, antisymmetric tensor field, or RR-field. In relation with string dynamics, U(1) gauge field on the brane has played an important role [8], of which dynamics is given by Born-Infeld type gauge kinetic term in the effective action on the brane [9, 10] since one of two U(1) gauge symmetries remains unbroken in the decaying D$\bar{\text{D}}$-system despite of tachyon condensation [11]. In pure rolling tachyon background, perturbative electromagnetic waves stop propagating around the tachyon vacuum as expected [12]. In the presence of both the rolling tachyon and a uniform electric field, there exists a critical value of the uniform electric field and perturbative waves on it propagate along electric flux lines, of which speed is given by magnitude of the background electric field [13, 14]. Though final remnants after decaying may be composed of uniformly stretched fundamental strings (F1's) and tachyon matter, dynamical process including confinement is not understood yet.

An intriguing question worth asking at the present stage is to look for general homogeneous configuration of the effective field theory of the tachyon with a runaway potential and Born-Infeld type U(1) gauge field, and to study properties of perturbative excitations on such background. For the system of unstable D3-brane, we showed that the most general homogeneous solution is the time-dependent rolling tachyon with constant electric and magnetic fields. Under a reasonable shape of the runaway tachyon potential, we find an exact form of the classical rolling tachyon solution, which may make further study more tractable in the scheme of effective field theory. A family of tachyonic vacua constructed at infinite vacuum expectation value of the tachyon is characterized by magnitude of the constant background electric and magnetic fields ($|\mathbf{E}_0|, |\mathbf{B}_0|$) and the angle θ between them. The configuration has nonvanishing momentum along the direction perpendicular to $\mathbf{E}_0$ and $\mathbf{B}_0$. By an appropriate Lorentz boost and a rotation we show that it is a pressureless fluid of fundamental strings made by turning on electric and magnetic fields and stretched along one direction in the rolling tachyon background, which will be called string fluid with electric and magnetic fields.

There is however a subtle point. Since the momentum density is nonvanishing in the frame that fields are homogeneous, it is not the rest frame of the whole configuration. In the rest frame obtained by a suitable Lorentz boost, the tachyon field is no longer homogeneous in the sense that $\partial_i T$ is not zero. In other words, the rest frame of the system and the frame where the tachyon field is homogeneous are different. This fact makes the configuration rather nontrivial and in general the electric field and the magnetic field are not completely (anti-)parallel though they form fluid of strings stretched along one direction. The angle θ between them may then be interpreted as encoding the relative motion between stretched strings along one direction and the rolling tachyon flowing in the perpendicular direction to the strings.

In order to reach the above interpretation we investigate the small fluctuations around the homogeneous configuration of the rolling tachyon and the electromagnetic fields ($|\mathbf{E}_0|$, $|\mathbf{B}_0|, \theta$). After a Lorentz boost and a rotation, the fluctuations are shown to propagate along the direction of the electric field with the propagation velocity equal to the electric field. Roughly, the whole situation in this frame is then as follows. The tachyon has a nonzero momentum since it is Lorentz boosted; the electric field and the magnetic field still make a nonzero angle and have their own momentum. Then these two contributions cancel each other, making the total momentum zero. This interpretation is also supported by the fact that the only nonzero components of the energy-momentum tensor are T_{00} and the diagonal component along the direction of the electric field.

This paper is organized as follows. In section 2, the most general homogeneous solution of rolling tachyon with constant electric and magnetic field is obtained. In section 3, propagation of small fluctuations on the obtained configuration is analyzed. Then via a suitable Lorentz boost and a rotation, we identify the background configuration as a fluid of strings with electric and magnetic fields stretched along the direction of electric field and rolling tachyon. In section 4, we conclude with discussion. Finally in appendix, detailed description of perturbative electromagnetic waves is given in the pure tachyon background.

2. Homogeneous configuration

In this section we introduce the system of tachyon coupled to an abelian gauge field and find the most general homogeneous solution which turns out to be constant electric and magnetic fields together with rolling tachyon configuration.

When we turn off antisymmetric tensor field of second rank in bosonic sector of effective action, the unstable D3-brane system is described by the following Born-Infeld type action [9, 10]

$$S = -T_3 \int d^4x \, V(T) \sqrt{-\det(\eta_{\mu\nu} + \partial_\mu T \partial_\nu T + F_{\mu\nu})}\,, \tag{2.1}$$

where T is tachyon and $F_{\mu\nu}$ is field strength tensor of abelian gauge field A_μ on the D3-brane.

Since tachyon potential $V(T)$ measures varying tension, it should satisfy two boundary values such that $V(T=0)=1$ and $V(T=\infty)=0$. Specific computation based on (boundary) string field theory [20, 10] gives $V(T) \sim e^{-T^2}$ and ref. [2] suggests $V(T) \sim e^{-T}$ for large T. Here we adopt a shape of the potential for the sake of convenient analysis

$$V(T) = \frac{1}{\cosh{(T/T_0)}}, \tag{2.2}$$

where T_0 is determined by string theory of our interest.

To proceed, we introduce a few notations. We first define

$$X_{\mu\nu} \equiv \eta_{\mu\nu} + \partial_\mu T \partial_\nu T + F_{\mu\nu}\,, \tag{2.3}$$

$$X \equiv \det(X_{\mu\nu})\,. \tag{2.4}$$

In $X_{\mu\nu}$, we separate barred metric $\bar{\eta}_{\mu\nu}$ and barred field strength tensor $\bar{F}_{\mu\nu}$

$$\bar{\eta}_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu T \partial_\nu T\,, \tag{2.5}$$

$$\bar{F}_{\mu\nu} = F_{\mu\nu}\,. \tag{2.6}$$

Then we have determinant of barred metric $\bar{\eta}$ and inverse metric $\bar{\eta}^{\mu\nu}$

$$\bar{\eta} = -(1 + \partial_\mu T \partial^\mu T)\,, \qquad \bar{\eta}^{\mu\nu} = \eta^{\mu\nu} - \frac{\partial^\mu T \partial^\nu T}{1 + \partial_\rho T \partial^\rho T}\,, \tag{2.7}$$

and contravariant barred field strength tensor $\bar{F}^{\mu\nu}$ and its dual field strength $\bar{F}^*_{\mu\nu}$

$$\bar{F}^{\mu\nu} = \bar{\eta}^{\mu\alpha}\bar{\eta}^{\nu\beta} F_{\alpha\beta}\,, \qquad \bar{F}^*_{\mu\nu} = \frac{\bar{\epsilon}_{\mu\nu\alpha\beta}}{2}\bar{F}^{\alpha\beta} = \frac{\bar{\epsilon}_{\mu\nu\alpha\beta}}{2}\bar{\eta}^{\alpha\gamma}\bar{\eta}^{\beta\delta}F_{\gamma\delta}\,, \tag{2.8}$$

where $\bar{\epsilon}_{\mu\nu\alpha\beta} = \sqrt{-\bar{\eta}}\;\epsilon_{\mu\nu\alpha\beta}$ with $\epsilon_{0123} = 1$.

In terms of barred quantities eq. (2.4) is computed as

$$X = \bar{\eta}\left[1 + \frac{1}{2}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} - \frac{1}{16}\left(\bar{F}^*_{\mu\nu}\bar{F}^{\mu\nu}\right)^2\right]. \tag{2.9}$$

Then equations of motion for the tachyon T and the gauge field A_μ are

$$\partial_\mu\left(\frac{V}{\sqrt{-X}}\, C^{\mu\nu}_{\rm S}\, \partial_\nu T\right) + \sqrt{-X}\,\frac{dV}{dT} = 0\,, \tag{2.10}$$

$$\partial_\mu\left(\frac{V}{\sqrt{-X}}\, C^{\mu\nu}_{\rm A}\right) = 0\,. \tag{2.11}$$

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Here $C_{\rm S}^{\mu\nu}$ and $C_{\rm A}^{\mu\nu}$ are symmetric and asymmetric part, respectively, of the cofactor,

$$C^{\mu\nu} = \bar{\eta}\left(\bar{\eta}^{\mu\nu} + \bar{F}^{\mu\nu} + \bar{\eta}^{\mu\alpha}\bar{\eta}^{\beta\gamma}\bar{\eta}^{\delta\nu}\bar{F}^{*}_{\alpha\beta}\bar{F}^{*}_{\gamma\delta} + \bar{\eta}^{\mu\alpha}\bar{\eta}^{\beta\gamma}\bar{F}^{*}_{\alpha\beta}\bar{F}^{*}_{\gamma\delta}\bar{F}^{\delta\nu}\right), \tag{2.12}$$

namely,

$$\begin{aligned} C_{\rm S}^{\mu\nu} &= \bar{\eta}(\bar{\eta}^{\mu\nu} + \bar{\eta}^{\mu\alpha}\bar{\eta}^{\beta\gamma}\bar{\eta}^{\delta\nu}\bar{F}^{*}_{\alpha\beta}\bar{F}^{*}_{\gamma\delta}), \\ C_{\rm A}^{\mu\nu} &= \bar{\eta}(\bar{F}^{\mu\nu} + \bar{\eta}^{\mu\alpha}\bar{\eta}^{\beta\gamma}\bar{F}^{*}_{\alpha\beta}\bar{F}^{*}_{\gamma\delta}\bar{F}^{\delta\nu}). \end{aligned} \tag{2.13}$$

Energy-momentum tensor $T_{\mu\nu}$ in a symmetric form is given by

$$T_{\mu\nu} = -\frac{T_3 V(T)}{\sqrt{-X}}\left[-\eta_{\mu\nu}X + \frac{1}{2}\left(C_{\mu\rho}(\partial_\nu T\partial^\rho T + F_\nu{}^\rho) + C_{\nu\rho}(\partial_\mu T\partial^\rho T + F_\mu{}^\rho)\right)\right], \tag{2.14}$$

where $C_{\mu\nu} \equiv \eta_{\mu\alpha}\eta_{\nu\beta}C^{\alpha\beta}$. Diagonal components can be identified as density ρ and averaged pressure p,

$$\rho = -\frac{T_3 V(T)}{\sqrt{-X}}\left[X + C_{0\mu}(\dot{T}\partial^\mu T + F_0{}^\mu)\right], \tag{2.15}$$

$$p = -\frac{T_3 V(T)}{3\sqrt{-X}}\left[-3X + C_{0\mu}(\dot{T}\partial^\mu T + F_0{}^\mu) + C^{\mu\nu}(\partial_\mu T\partial_\nu T + F_{\mu\nu})\right]. \tag{2.16}$$

From the equation of state $p = w\rho$, we read

$$w = \frac{1}{3} - \frac{4X - C^{\mu\nu}(\partial_\mu T\partial_\nu T + F_{\mu\nu})}{3\left[X + C_{0\mu}(\dot{T}\partial^\mu T + F_0{}^\mu)\right]}. \tag{2.17}$$

Thus trace of the energy-momentum tensor (2.14) is given by

$$T^\mu_{\ \mu} = -\frac{T_3 V(T)}{\sqrt{-X}}\left[(4 + 3\partial_\rho T\partial^\rho T) + \left(\left(1 + \frac{1}{2}\partial_\rho T\partial^\rho T\right)\delta^\mu_{\ \sigma} - \partial^\mu T\partial_\sigma T\right)F_{\mu\nu}F^{\sigma\nu}\right]. \tag{2.18}$$

Let us consider homogeneous configurations of $T(t)$, $\mathbf{E}(t)$, and $\mathbf{B}(t)$ defined by $E^i = F_{i0}$ and $B^i = \epsilon_{0ijk}F_{jk}/2$. Then Faraday's law

$$\nabla\times\mathbf{E} + \frac{\partial\mathbf{B}}{\partial t} = 0 \tag{2.19}$$

allows only constant magnetic field $\mathbf{B} = \mathbf{B}_0$. The equations of motion (2.10) and (2.11) become

$$\partial_0\left(\frac{V}{\sqrt{-X}}\dot{T}\right) + \frac{\sqrt{-X}}{1+\mathbf{B}_0^2}\frac{dV}{dT} = 0, \tag{2.20}$$

$$\left(\delta_{ij} + B_0^i B_0^j\right)\partial_0\left(\frac{V}{\sqrt{-X}}E^j\right) = 0, \tag{2.21}$$

where

$$X = -(1-\dot{T}^2)(1+\mathbf{B}^2) + \mathbf{E}^2 + (\mathbf{E}\cdot\mathbf{B})^2. \tag{2.22}$$

The energy density ρ (2.15) reduces to

$$\rho = T_3 \frac{V}{\sqrt{-X}} (1 + \mathbf{B}_0^2) , \tag{2.23}$$

and is a constant, which can be seen from the equations (2.20)–(2.21) as it should be. Eq. (2.21) then becomes

$$(\delta^{ij} + B_0^i B_0^j) \dot{E}^j = 0 . \tag{2.24}$$

Since the matrix $\delta^{ij} + B_0^i B_0^j$ is positive definite, eq. (2.24) allows only constant electric field $E^i = E_0^i$. Therefore, the most general homogeneous solution of eqs. (2.20)–(2.21) is $\mathbf{E} = \mathbf{E}_0 =$ constant, $\mathbf{B} = \mathbf{B}_0 =$ constant, and $V/\sqrt{-X} =$ constant. When $\mathbf{E}_0$ and $\mathbf{B}_0$ are not parallel or anti-parallel to each other, this configuration has nonvanishing linear momentum density

$$\mathcal{P}_i = T^{0i} = T_3 \frac{V}{\sqrt{-X}} \epsilon_{ijk} E_0^j B_0^k , \tag{2.25}$$

so does angular momentum density.

Since ρ is a constant, X must vanish at the minimum of the tachyon potential $V(T \to \infty) = 0$, and it determines $\dot{T}_\infty = \dot{T}(T \to \infty)$

$$\dot{T}_\infty^2 = 1 - \frac{\mathbf{E}_0^2 + (\mathbf{E}_0 \cdot \mathbf{B}_0)^2}{1 + \mathbf{B}_0^2} . \tag{2.26}$$

Thus, the solution is specified by three parameters, namely, the magnitudes of $\mathbf{E}_0$ and $\mathbf{B}_0$, and the angle θ between them. When $\mathbf{B}_0 = 0$, eq. (2.26) coincides trivially with the result of pure electric case in ref. [13, 14]. If $\mathbf{E}_0 = 0$, the presence of $\mathbf{B}_0 \neq 0$ plays no role in the equation of motion (2.20) except for a constant overall scale. When $\mathbf{E}_0 \neq 0$ and $\mathbf{B}_0 \neq 0$, we consider the set of allowed values of $(|\mathbf{E}_0|, |\mathbf{B}_0|, \theta)$ for given initial values T_i and $\dot{T}_i$ of T and $\dot{T}$, satisfying $-X > 0$. Then, the energy density (2.23) has maximum for both parallel ($\theta = 0$) and anti-parallel ($\theta = \pi$) cases, and minimum for orthogonal case ($\theta = \pi/2$). For a given angle θ, the energy density ρ is monotonically-increasing function of both $|\mathbf{E}_0|$ and $|\mathbf{B}_0|$ as expected. When $\mathbf{E}_0$ is parallel or anti-parallel to $\mathbf{B}_0$, $|\mathbf{E}_0|$ has upper bound $\sqrt{1 - \dot{T}_i^2}$ irrespective of $|\mathbf{B}_0|$. When $\mathbf{E}_0$ is orthogonal to $\mathbf{B}_0$, upper limit of $|\mathbf{E}_0|$ is increased as $|\mathbf{B}_0|$ increases so that a configuration of infinite $|\mathbf{E}_0|$ and $|\mathbf{B}_0|$ satisfying $\mathbf{E}_0^2 - \mathbf{B}_0^2 = -F_{\mu\nu}^0 F_0^{\mu\nu}/2 \leq 1$ is available.

One may wonder whether the solutions $(|\mathbf{E}_0|, |\mathbf{B}_0|, \theta)$ are not all independent but related by some Lorentz transformations which could change the angle θ, for example. However this is not the case since, under a Lorentz boost, the tachyon field T is no longer homogeneous in space. Therefore in general they describe physically distinct configurations. To be more explicit, let us work in the coordinate where the electric field is directed in the x-direction and the magnetic field lies on the xy-plane,

$$\mathbf{E}_0 = E_0 \hat{\mathbf{x}} , \qquad \mathbf{B}_0 = B_1 \hat{\mathbf{x}} + B_2 \hat{\mathbf{y}} , \tag{2.27}$$

where $B_1 = |\mathbf{B}_0| \cos\theta$ and $B_2 = |\mathbf{B}_0| \sin\theta$. This configuration has the momentum density $\mathcal{P}_3$ in the z-direction as seen in eq. (2.25). Other nonvanishing components of the energy-momentum tensor (2.14) are

$$\begin{aligned}
T_{11} &= -T_3 \frac{V}{\sqrt{-X}}(1-\dot{T}^2)(1+B_1^2)\,, \\
T_{22} &= -T_3 \frac{V}{\sqrt{-X}}\left[(1-\dot{T}^2)(1+B_2^2)-E_0^2\right], \\
T_{33} &= -T_3 \frac{V}{\sqrt{-X}}(1-\dot{T}^2-E_0^2)\,, \\
T_{12} &= -T_3 \frac{V}{\sqrt{-X}}(1-\dot{T}^2)B_1B_2\,.
\end{aligned} \tag{2.28}$$

We will discuss more about this homogeneous configuration in section 3 when small fluctuations around it are considered.

Let us turn to rolling of the tachyon condensate in the presence of uniform electric and magnetic fields. For the tachyon potential (2.2), from constancy of the energy density, we have a solution

$$T(t) = T_0 \sinh^{-1}\left(a_+ e^{(\dot{T}_\infty/T_0)\,t} - a_- e^{-(\dot{T}_\infty/T_0)\,t}\right), \tag{2.29}$$

where $a_\pm = \frac{1}{2}\left[(\dot{T}_i/\dot{T}_\infty)\cosh(T_i/T_0) \pm \sinh(T_i/T_0)\right]$. Compared to the case of no electromagnetic field for which $\dot{T}_\infty = 1$, the solution (2.29) has rescalings of the energy density $\rho \to \rho\left[(1+\mathbf{B}_0^2)(1-\dot{T}_i^2)/(\dot{T}_\infty^2-\dot{T}_i^2)\right]^{1/2}$ and $T_0 \to T_0/\dot{T}_\infty$ in the exponents of eq. (2.29). As the value of $[\mathbf{E}_0^2 + (\mathbf{E}_0\cdot\mathbf{B}_0)^2]/(1+\mathbf{B}_0^2)$ approaches unity, $\dot{T}_\infty$ almost vanishes from eq. (2.26) and not only the energy density (2.23) becomes larger, but also the time scale of remaining on top of the tachyon potential becomes longer. This may be a nice feature for various aspects in cosmology, e.g., the rolling tachyon as a source of sufficient inflation [4]–[7] and [15]. In trials to use pure tachyon rolling as a source of inflation, sufficient inflation could not be obtained with the potential derived from string theory and *ad hoc* potentials had to be introduced. With the tachyon in the gauge field background, sufficient inflation may be obtained near the top of the tachyon potential. However, a difficulty in applying unstable D-branes with electric and magnetic flux to cosmology is that the constant electric and magnetic field background is homogeneous, but not isotropic. Therefore, the electric and magnetic fields must disappear at the end of inflation.

3. Small fluctuations and electromagnetic string fluid

In this section, we investigate the propagation of small fluctuations in the homogeneous background field discussed in the previous section. Through the analysis we will argue that the homogeneous configuration is identified as a fluid consisting of strings carrying electric and magnetic fields stretched along one direction and the rolling tachyon flowing in a perpendicular direction to the strings. The relative velocity between strings and the rolling tachyon is essentially encoded in the angle θ between the electric field and the magnetic field.

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In the pure rolling tachyon background, propagation of small fluctuations is completely suppressed [12], meaning the absence of perturbative degrees of freedom. In ref. [13, 14], it was shown that with the electric field turned on 1+1 dimensional propagation of small fluctuations is allowed along the electric flux lines. Here we first extend this analysis including the magnetic field in unstable D3-branes.

In the rolling tachyon limit $\dot{T} \to \dot{T}_\infty$, with the constant electric and magnetic field background, the equations of motion of the gauge fields and the tachyon are combined into[1]

$$(\eta + F)^{-1\,(AB)} \partial_A F_{BC} = 0\,, \tag{3.1}$$

where indices $A, B, \ldots = 0, \ldots, 3, T$ with $F_{\mu T} = -F_{T\mu} = \partial_\mu T$ and (AB) means the symmetric part of $(\eta + F)^{-1}$. The fields are divided into the background F^0_{AB} and the small fluctuations f_{AB}

$$F_{AB} = F^0_{AB} + f_{AB}\,. \tag{3.2}$$

We choose the frame where the background fields are homogeneous and work in the background (2.27) considered in section 2. Then the background F^0 takes the form

$$F^0 = \begin{pmatrix} 0 & -E_0 & 0 & 0 & \dot{T} \\ E_0 & 0 & 0 & -B_2 & 0 \\ 0 & 0 & 0 & B_1 & 0 \\ 0 & B_2 & -B_1 & 0 & 0 \\ -\dot{T} & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{3.3}$$

In the rolling tachyon limit, $\dot{T} = \dot{T}_\infty = \sqrt{1 - E_0^2(1 + B_1^2)/(1 + \mathbf{B}_0^2)}$. The fluctuations f_{AB} can be written in terms of the gauge potential a_μ and the tachyon fluctuation $\hat{t}$ as $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ and $f_{\mu T} = \partial_\mu \hat{t}$. We will work in Weyl gauge $a_0 = 0$ and define

$$\begin{aligned} \partial'_0 &= \frac{(1 + \mathbf{B}_0^2)\partial_0 + E_0 B_2 \partial_3}{\sqrt{(1 + \mathbf{B}_0^2)^2 - E_0^2 B_2^2}}\,, \\ \partial'_1 &= \frac{(1 + B_1^2)\partial_1 + B_1 B_2 \partial_2}{\sqrt{(1 + B_1^2)^2 + B_1^2 B_2^2}}\,, \\ v'^2 &= \frac{E_0^2\left[(1 + B_1^2)^2 + B_1^2 B_2^2\right]}{(1 + \mathbf{B}_0^2)^2 - E_0^2 B_2^2} \le 1\,, \end{aligned} \tag{3.4}$$

where the inequality in the last line comes from eq. (2.26). Then the equations of motion of the fluctuations derived from eq. (3.1) are

$$E_0 \partial_0 G = 0\,, \tag{3.5}$$

$$\partial_0'^2 a_i - v'^2 \partial_1'^2 a_i - \frac{E_0}{(1 + \mathbf{B}_0^2)^2 - E_0^2 B_2^2} \partial_i G = 0\,, \tag{3.6}$$

$$\partial_0'^2 \hat{t} - v'^2 \partial_1'^2 \hat{t} = 0\,, \tag{3.7}$$

[1] We neglected the potential part since we are mainly interested in the causal behavior [14].

where

$$G = \sqrt{(1+B_1^2)^2 + B_1^2 B_2^2}\,\partial_1' \left[\sqrt{(1+\mathbf{B}_0^2)^2 - E_0^2(1+\mathbf{B}_0^2)(1+B_1^2)}\,\hat{t} - \right.$$
$$\left. - E_0\left((1+B_1^2)a_1 + B_1 B_2 a_2\right)\right] +$$
$$+ \sqrt{(1+\mathbf{B}_0^2)^2 - E_0^2 B_2^2}\; B_2 \partial_0' a_3\,. \tag{3.8}$$

The first equation (3.5) implies $G = G(\mathbf{x})$. Choice of the $a_0 = 0$ gauge leaves us the space-dependent gauge transformation $\Lambda(\mathbf{x})$ and under this gauge transformation $G(\mathbf{x})$ transforms to $G(\mathbf{x}) - E_0[((1+B_1^2)\partial_1 + B_1 B_2 \partial_2)^2 - B_2^2 \partial_3^2]\Lambda(\mathbf{x})$. Thus $G(\mathbf{x})$ is not a propagating mode and we can set $G(\mathbf{x}) = 0$ as a gauge choice. Now there remain three propagating modes which are mixtures of a_i and $\hat{t}$, and obey the wave equation of the form

$$\partial_0'^2 a - v'^2 \partial_1'^2 a = 0\,. \tag{3.9}$$

Therefore, the fluctuations propagate along only one direction, x-direction in the primed frame, and v' is the propagating speed in that frame. The Lorentz transformation connecting the homogeneous frame to the primed frame is composed of a rotation in the xy-plane by

$$\varphi = \tan^{-1}\left(\frac{B_1 B_2}{1+B_1^2}\right) = \tan^{-1}\left(\frac{\mathbf{B}_0^2 \cos\theta \sin\theta}{1+\mathbf{B}_0^2\cos^2\theta}\right) \tag{3.10}$$

and a boost along z-direction by $\beta = E_0 B_2/(1+\mathbf{B}_0^2)$. Propagation velocity $\mathbf{v}$ in the homogeneous frame is given by

$$\mathbf{v} = (\sqrt{1-\beta^2}\,v'\cos\varphi, \sqrt{1-\beta^2}\,v'\sin\varphi, \beta)\,, \tag{3.11}$$

and the speed is

$$v^2 = \beta^2 + (1-\beta^2)\,v'^2 = \frac{\mathbf{E}_0^2 + (\mathbf{E}_0\cdot\mathbf{B}_0)^2}{1+\mathbf{B}_0^2} = 1 - \dot{T}_\infty^2\,. \tag{3.12}$$

Note that it is nothing but an expression in the vacuum condition (2.26). For fixed E_0 and $\mathbf{B}_0$, the velocity v has the maximum value E_0 when $\mathbf{E}_0$ and $\mathbf{B}_0$ are parallel or anti-parallel, and the minimum value $E_0/\sqrt{1+\mathbf{B}_0^2}$ when they are orthogonal. The propagation direction angle φ in the xy-plane increases as the angle θ between the electric and magnetic fields increases from zero, reaching a maximum value $\varphi_{\max} = \tan^{-1}(\mathbf{B}_0^2/2\sqrt{1+\mathbf{B}_0^2})$ at $\theta = \cos^{-1}(1/\sqrt{2+\mathbf{B}_0^2})$, then decreases back to zero at $\theta = \pi/2$. For any value of $|\mathbf{B}_0|$, we have $\varphi < \theta$ for $0 < \theta \le \pi/2$. For $\pi/2 < \theta < \pi$, φ takes a negative value $-\varphi(\pi-\theta)$ which keeps $|\varphi| < \theta - \pi/2$. The direction of propagation coincides with that of $|\mathbf{B}_0|$ in the limit $|\mathbf{B}_0|\cos\theta \to \infty$.

Since the wave equation of the fluctuations has the standard form (3.9) in the primed frame, it would be illuminating to reconsider the configuration in the primed frame. After the Lorentz boost and the rotation considered above, background gauge fields are transformed to

$$\begin{aligned}
\mathbf{E}'_0 &\equiv E'_0\hat{\mathbf{x}}' = v'\hat{\mathbf{x}}' \,, \\
\mathbf{B}'_0 &\equiv B'_1\hat{\mathbf{x}}' + B'_2\hat{\mathbf{y}}' \\
&= \frac{E_0}{v'}\left[B_1\hat{\mathbf{x}}' + \frac{\dot{T}^2_\infty(1+\mathbf{B}_0^2)}{(1+\mathbf{B}_0^2)^2 - E_0^2B_2^2}B_2\hat{\mathbf{y}}'\right] .
\end{aligned} \tag{3.13}$$

Also the energy-momentum tensor in the primed frame can be obtained from eq. (2.28),

$$\begin{aligned}
T'_{00} &= T_3\frac{V}{\sqrt{-X}}\left[\frac{(1+\mathbf{B}_0^2)^2 - E_0^2B_2^2}{1+\mathbf{B}_0^2} - \frac{E_0^2B_2^2}{(1+\mathbf{B}_0^2)^2 - E_0^2B_2^2}(\dot{T}^2_\infty - \dot{T}^2)\right] , \\
T'_{11} &= -v'^2T'_{00} - T_3\frac{V}{\sqrt{-X}}\frac{(1+B_1^2)^3 + B_1^2B_2^2(2B_1^2+B_2^2+3)}{(1+B_1^2)^2 + B_1^2B_2^2}(\dot{T}^2_\infty - \dot{T}^2) \,, \\
T'_{03} &= -T_3\frac{V}{\sqrt{-X}}\frac{E_0B_2(1+\mathbf{B}_0^2)}{(1+\mathbf{B}_0^2)^2 - E_0^2B_2^2}(\dot{T}^2_\infty - \dot{T}^2) \,, \\
T'_{22} &= -T_3\frac{V}{\sqrt{-X}}\frac{(1+B_1^2)(1+\mathbf{B}_0^2)}{(1+B_1^2)^2 + B_1^2B_2^2}(\dot{T}^2_\infty - \dot{T}^2) \,, \\
T'_{33} &= -T_3\frac{V}{\sqrt{-X}}\frac{(1+\mathbf{B}_0^2)^2}{(1+\mathbf{B}_0^2)^2 - E_0^2B_2^2}(\dot{T}^2_\infty - \dot{T}^2) \,, \\
T'_{12} &= -T_3\frac{V}{\sqrt{-X}}\frac{B_1B_2(1+\mathbf{B}_0^2)}{(1+B_1^2)^2 + B_1^2B_2^2}(\dot{T}^2_\infty - \dot{T}^2) \,.
\end{aligned} \tag{3.14}$$

Then, in the limit $\dot{T} \to \dot{T}_\infty$, the only nonvanishing components are T'_{00} and T'_{11}.

Interpretation of the configuration is now more clear in the primed frame. Note that T'_{22} and T'_{33} vanish and hence there is no pressure in y' and z directions; the final configuration of the rolling tachyon limit may then be considered as a pressureless fluid of one-dimensional objects stretched along x'-direction. Also, the momentum density T'^{0i} vanishes in this frame, meaning that this frame corresponds to the rest frame of the string fluid background which consists of the homogeneous gauge fields and the rolling tachyon. Of course this is consistent with the fact that the equations for small fluctuations have the standard form (3.9) in the primed frame. Moreover, the propagation velocity $\mathbf{v}'$ is nothing but the same as the electric field $\mathbf{E}'_0$ in the rest frame of the fluid.

When there is no magnetic field $\mathbf{B}_0 = 0$, such a one-dimensional object is often identified as the fundamental string [16, 8, 13, 14]. In the present case that $\mathbf{B}_0 \neq 0$, one may also interpret the object as a sort of string with electric and magnetic fields though more detailed string-theory analysis including confinement mechanism is needed to draw any definite conclusion. Note, however, that $\mathbf{E}'_0$ and $\mathbf{B}'_0$ are not in general (anti-)parallel in eq. (3.13). This can be understood roughly in the following way. In the Lorentz-boosted primed frame, the tachyon field is no longer homogeneous in the sense that $\partial_i T$ is not zero but it has linear momentum in z-direction, $T = T((t' - \beta z)/\sqrt{1-\beta^2})$. Then the contribution of the gauge field should cancel the momentum of the tachyon since total momentum density is zero.

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This is possible if the electric and the magnetic field make a nonzero angle, which is realized by boosting parallel electric and magnetic field configurations. This interpretation is also consistent with the facts that B_2' vanishes when $\dot{T}_\infty$ goes to zero in eq. (3.13).

Now the whole picture of the rolling tachyon in electromagnetic background is as follows. Initially the unstable D3-brane has constant electric and magnetic fields with an arbitrary angle. As brane decays the electric field and the magnetic field form a pressureless fluid of strings. Also, when they are not parallel, part of the energy is used to boosts the strings relative to the homogeneous rolling tachyon field, rather than the entire energy is exhausted in the formation of string-like object or lower dimensional D-branes. Though the brane-decaying process would occur rather violently instead of occurring through homogeneous tachyon rolling, one might still think of the expression of the energy-momentum tensor in eq. (3.14) as roughly representing how the formation and boost of the string fluid is realized as $\dot{T}$ approaches $\dot{T}_\infty$.

The final configuration is then a fluid consisting of string with electric and magnetic fields stretched along x'-directions and the rolling tachyon moving in z-direction relative to the strings. The rest frame of the whole configuration is the primed frame defined above in which small fluctuations propagate along the x'-direction with speed $v' = |\mathbf{E}_0'|$. Note however that the fluid is not a noninteracting sum of strings and the rolling tachyon; it is rather a nontrivial composite, since the gauge fluctuations and the tachyon fluctuation do not separate but are mixed together as seen above. Since this is due to the effect of inhomogeneous configuration of the tachyon field in the rest frame of the fluid, it would be interesting to study the consequence of more nontrivial inhomogeneous rolling tachyon configurations [17].

4. Conclusion and discussion

We have seen that effective field theory of unstable D3-brane system, composed of a tachyon field and an abelian gauge field, supports time-dependent rolling tachyon configuration with constant electric and magnetic fields as the most general homogeneous solution. Tachyonic vacua are labeled by magnitudes of electric and magnetic fields and the angle between them. Through the analysis of small fluctuations in this background we find that the final configuration in the rolling tachyon limit is interpreted as a fluid consisting of strings with electric and magnetic fields stretched along one direction and the rolling tachyon moving in a perpendicular direction to the strings. The fluctuations propagate along the direction of strings with velocity equal to the electric field.

Though we considered the case of unstable D3-brane, it would be interesting to understand Dp-branes of arbitrary dimensions. For higher dimensional Dp-branes, further study is needed because determinant in the Born-Infeld type action contains higher-order terms and magnetic components of the field strength tensor become more complicated such as $B_{i_1 i_2 \cdots i_{p-2}} = \epsilon_{0 i_1 i_2 \cdots i_p} F_{i_{p-1} i_p}/2$. However, once the objects are generated, then they are likely to be higher dimensional analogue of the strings with electric and magnetic fields. When $p = 2$, magnetic field has single component $B = \epsilon_{012} F_{12}$ and the determinant in the

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Born-Infeld type action is quadratic, and hence only the orthogonal case is realized. This kind of analysis should also be addressed in the scheme of string field theory [2, 13, 17, 19].

It would be interesting if the following points are also understood. Though our analysis has been accomplished by adopting the specific action from ref. [9, 2, 20], vacuum structure and propagation of electromagnetic field probably share universal behavior even in general BSFT type action [10] due to the simple limiting property of kinetic term of the BSFT actions as argued in ref. [12, 14]. In section 3 and appendix, we utilized quadratic expansion for fluctuations of the tachyon and the gauge field to identify mainly stringy objects and their propagating velocity, however description of mutual interaction between fluctuating degrees requires at least quartic expansion or some nonperturbative approaches.

Addition of other bosonic degrees are also worth tackling. Inclusion of gravity may let us ask what the gravitating version of the stretched strings in the rolling tachyon background is. If we take into account the rolling tachyon as a viable source of inflationary universe, then photons of Born-Infeld type action on the unstable branes can also be testified as a component of materializing radiation dominated era. In such sense existence of constant electromagnetic background field seems beneficial for boosting generation of the photons even in late time, but direct application is difficult due to the lack of isotropy. Taking into account gravitational radiation is also intriguing to understand the fate of tachyon matter [21]. When we can deal with constant background antisymmetric tensor field $B_{\mu\nu}$ which is the same as constant background electromagnetic field $(\mathbf{E}_0, \mathbf{B}_0)$ in the action on the brane, distinction between them can easily be tested by pattern of propagating small fluctuations in the given background.

Note added. After completion of this work, ref. [23] appeared on the archive, which considered the case that electric and magnetic fields are orthogonal in the context of both string and effective theories.

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A. Electromagnetism in the pure tachyon background

In this appendix we will discuss in detail the fluctuations around rolling tachyon background (2.29) without electric or magnetic field, $\mathbf{E}_0 = \mathbf{B}_0 = 0$.

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The fluctuation $\delta T(t,\mathbf{X})$ of tachyon satisfies the linearized equation

$$\partial_0\left(\frac{\delta\dot{T}}{1-\dot{T}^2}\right)-\nabla^2\delta T+\frac{d^2\ln V}{dT^2}\delta T=0\,, \tag{A.1}$$

and it decouples from the electromagnetic fluctuations. Near the top of the potential it reduces to the Klein-Gordon equation with negative mass and diverges exponentially as it should. At late time, $\dot{T}\to 1$, the first term dominates and the fluctuation stops propagating [2]. As we will see shortly, the fluctuation of the gauge field also shows the same behavior. Actually this is a consequence of carrollian limit of vanishing light speed at late time of rolling tachyon without background electromagnetic field [12, 13, 14].

Now we consider the propagation of the gauge field A_μ in the rolling tachyon background (2.29) by linearizing the equation of motion (2.11) in this background. Written in terms of electric and magnetic fields, it becomes a modified Ampére's law

$$\nabla\times\mathbf{B}=\frac{\epsilon_0^2}{V(t)^2}\frac{\partial\mathbf{E}}{\partial t}\,, \tag{A.2}$$

where we have used the fact that $V(T)/\sqrt{1-\dot{T}^2}\equiv\epsilon_0$ is a constant. Then one may identify the tachyon potential $V(t)/\epsilon_0$ as the varying speed of electromagnetic waves. In particular, the late time behavior $V(t)\to 0$ corresponds to the carrollian limit. This seems to suggest a dictionary 'velocity of electromagnetic waves and tachyon potential' like 'time and tachyon' in ref. [22]. However, it should be confirmed by careful analysis.

Combined with the Faraday's law (2.19), the wave equations of the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ can be written as

$$\frac{\epsilon_0^2}{V^2}\frac{\partial^2\mathbf{B}}{\partial t^2}-\nabla^2\mathbf{B}=0\,, \tag{A.3}$$

$$\frac{\partial}{\partial t}\left(\frac{\epsilon_0^2}{V^2}\frac{\partial\mathbf{E}}{\partial t}\right)-\nabla^2\mathbf{E}=0\,. \tag{A.4}$$

Note that these equations are not symmetrical since the Ampére's law (A.2) is modified but Faraday's law (2.19) is left unchanged. More specifically, eq. (A.4) has a kind of damping term which is the first order in time derivative and the propagation of electric field shows rather different behavior from that of magnetic field as demonstrated below.

To investigate in detail the propagation of electromagnetic wave in the rolling tachyon background, let us try to find a plane-wave solution of Eqs. (A.3) and (A.4). Without loss of generality we consider the plane-wave propagating along the x-axis, $\mathbf{k}=k\hat{\mathbf{x}}$, and make an ansatz $\mathbf{B}=B_z(t,x)\hat{\mathbf{z}}$ with

$$B_z(t,x)=B(t)e^{ik[x-W_B(t)]}\,, \tag{A.5}$$

where $B(t)$ and $W_B(t)$ are real. We assume that $V(0)\approx 1$ and $\dot{W}_B(0)\approx 1$ so that the ansatz (A.5) gives the normal plane-wave initially. Since $\mathbf{E}$ and $\mathbf{B}$ are related through the Ampére's law (A.2) and Faraday's law (2.19), $\mathbf{E}=E_y(t,x)\hat{\mathbf{y}}$ is then given by

$$E_y(t,x)=E(t)e^{ik[x-W_E(t)]}\,, \tag{A.6}$$

with

$$E = \sqrt{\dot{W}_B^2 B^2 + k^{-2}\dot{B}^2}\,, \tag{A.7}$$

$$W_E = W_B - k^{-1}\tan^{-1}\left(\frac{\dot{B}}{k\dot{W}_B B}\right). \tag{A.8}$$

With this ansatz, eq. (A.3) reduces to

$$\ddot{B} - k^2\left(\dot{W}_B^2 - \epsilon_0^{-2}V^2\right)B = 0\,, \tag{A.9}$$

$$2\dot{B}\dot{W}_B + B\ddot{W}_B = 0\,. \tag{A.10}$$

Eq. (A.10) implies

$$\dot{W}_B = \frac{1}{\epsilon_0}\left(\frac{B_0}{B}\right)^2\,, \tag{A.11}$$

where B_0 is a constant. From eq. (A.8) we also find

$$\dot{W}_E = \frac{V^2}{\epsilon_0^3}\left(\frac{B_0}{E}\right)^2\,. \tag{A.12}$$

Inserting eq. (A.11) into eq. (A.9), we obtain a second-order nonlinear equation for amplitude of the magnetic field

$$\ddot{B} - \frac{k^2}{\epsilon_0^2}\left[\left(\frac{B_0}{B}\right)^4 - V^2\right]B = 0\,. \tag{A.13}$$

On the top of the potential $V = 1$, we get a normal plane-wave solution $\dot{W}_B = \dot{W}_E = 1/\epsilon_0$ with amplitudes of $B = B_0$ and $E = B_0/\epsilon_0$ as expected from the original wave equations (A.3)–(A.4).

To find more detailed behavior of the solution, we need to know the explicit form of $V(T(t))$. With the tachyon potential (2.2) and the tachyon background (2.29), it becomes

$$V(T(t)) = \frac{1}{\sqrt{1 + \left(a_+ e^{t/T_0} - a_- e^{-t/T_0}\right)^2}}\,. \tag{A.14}$$

In this case we solved eq. (A.13) numerically and the result is shown in figure 1. Note that the electric field goes to a constant value, while the magnetic field linearly diverges. Also both $\dot{W}_E$ and $\dot{W}_B$ vanish after some time, which means that electromagnetic wave freezes and does not propagate after the tachyon rolling. In addition, one can see from the last figure of figure 1 that waves with short wavelengths are frozen earlier than those with long wavelengths. Finally $\dot{W}_E$ tends to vanish faster than $\dot{W}_B$, i.e., the electric field is frozen more quickly than the magnetic field. This is because $\dot{W}_E$ in (A.12) contains extra V^2 compared with $\dot{W}_B$ in eq. (A.11).

For the onset behavior, one can also obtain analytical expressions. Once the tachyon rolls from the top of the potential, the fluctuations of the gauge fields start deviating from

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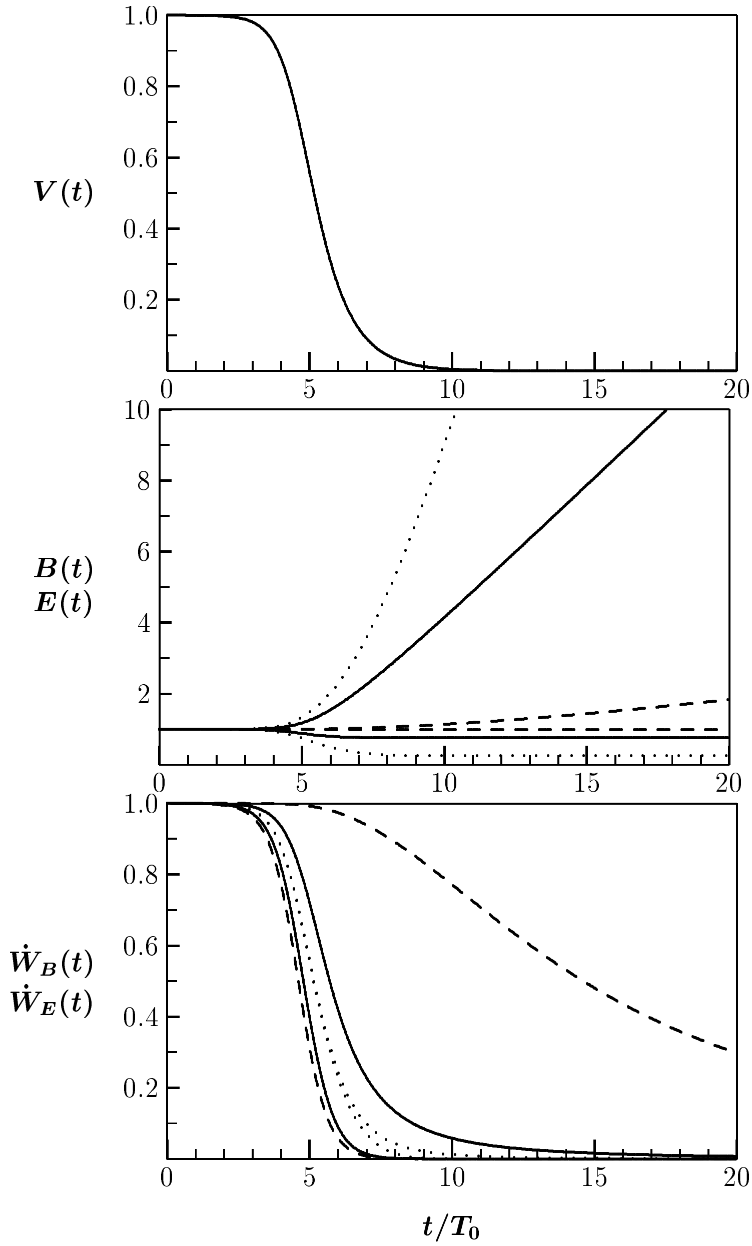

Figure 1: (Top) tachyon potential as a function of time, eq. (A.14) for $a_+ = 0.1$ and $a_- = 0$ ($\epsilon_0 = 1$). (Middle) amplitudes and (bottom) propagation speeds of magnetic field (upper three lines) and electric field (lower three lines) for $kT_0 = 0.1$ (dashed line), $kT_0 = 1$ (solid line), and $kT_0 = 10$ (dotted line).

the normal plane-wave solution. With the initial condition $V(t=0) = 1$, that is $T_i = 0$, $V(T(t))$ reduces to

$$V(T(t)) = \frac{1}{\sqrt{1 + \dot{T}_i \sinh^2(t/T_0)}} \,. \tag{A.15}$$

Assuming that $\dot{T}_i$ is small, which means the tachyon rolls slowly initially, we can solve

Eqs. (A.13) and (A.11) to the first-order in $\dot{T}_i^2$:

$$B(t) = B_0 \left[1 + \frac{\dot{T}_i^2 \, k^2 T_0^2}{4(1+k^2T_0^2)} \sinh^2\left(\frac{t}{T_0}\right) \right],$$
$$W_B(t) = t - \frac{\dot{T}_i^2}{4} \left[t + \frac{k^2 T_0^3}{2(1+k^2T_0^2)} \sinh\left(\frac{2t}{T_0}\right) \right], \tag{A.16}$$

where we have rescaled B_0 so that $B(0) = B_0$. This solution is valid as long as $\dot{T}_i^2 e^{t/T_0} \ll 1$. Thus the amplitude initially grows quadratically as the tachyon starts rolling. On the other hand, electric field can be calculated from eqs. (A.7) and (A.12),

$$E(t) = B_0 \left[1 - \frac{\dot{T}_i^2}{4(1+k^2T_0^2)} \left(1 + 2k^2T_0^2 + k^2T_0^2 \sinh^2\left(\frac{t}{T_0}\right)\right) \right],$$
$$W_E(t) = t - \frac{\dot{T}_i^2}{8(1+k^2T_0^2)} \left[(4+6k^2T_0^2)t + (2+k^2T_0^2)\sinh\left(\frac{2t}{T_0}\right) \right]. \tag{A.17}$$

Note that $E(t)$ decreases in contrast with the behavior of magnetic field and $\dot{W}_E$ decreases faster than $\dot{W}_B$. This can be confirmed in figure 1.

At late time, the value of the potential (A.14) decays exponentially to zero

$$V(T(t)) \approx \frac{1}{a_+ e^{t/T_0}}, \tag{A.18}$$

so that eq. (A.13) becomes, up to $\mathcal{O}(e^{-2t/T_0})$,

$$\ddot{B} - \frac{k^2 B_0^4}{\epsilon_0^2} \frac{1}{B^3} = 0, \tag{A.19}$$

The solution of this equation is given by

$$B(t) = \left[\left(\frac{k^2 B_0^4}{\epsilon_0^2 b_0^2} + \dot{b}_0^2 \right) t^2 + 2 b_0 \dot{b}_0 t + b_0^2 \right]^{1/2} + \mathcal{O}(e^{-t/T_0}), \tag{A.20}$$

where b_0 and $\dot{b}_0$ are constants. Therefore $B(t)$ increases linearly in time and then the propagation speed decreases according to eq. (A.11) (See also figure 1). Since B linearly increases, our linear approximation of the equation of motion is not valid as t becomes large. However, one may still extract some relevant physics as the tachyon rolls to the vacuum, $V \to 0$. The electromagnetic waves become frozen and stop propagating. For the electric field in this limit, we find that it approaches a constant value given by eq. (A.7). Of course, this is an expected behavior since one may interpret the infinite time limit of the rolling tachyon as the carrollian limit. That is, the effective covariant metric tensors $\bar{\eta}_{\mu\nu} = \mathrm{diag}(-1+\dot{T}^2, 1, 1, 1)$ (2.5) goes to $\mathrm{diag}(0,1,1,1)$, and light cone defined from $\bar{\eta}_{\mu\nu}$ collapses down to a timelike half line [14].

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